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An analysis of film boiling of a binary mixture in a porous medium

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Abstract—A theoretical analysis is performed to outline the effect of diffusion phenomena on the film boiling heat transfer process from a vertical plate to a liquid mixture in a porous medium. With boundary layer approximations, the conservation laws are resolved by means of similarity transformations. The resulting set of differential equations is solved using a shooting technique. Heat transfer results are obtained for a range of operating conditions applied to a mixture of water and methanol. Experiments are conducted to verify the theoretical findings, and excellent agreement is found between theory and experiment.

INTRODUCTION

Two phase flows of mixtures in a porous medium have numerous important engineering applications, from geothermal energy utilization, oil recovery, to nuclear and chemical processes. Furthermore, since in many industrial applications, the fluids of interest are not necessarily single component fluids, phase change of mixtures constitutes a broad area of research from a heat transfer point of view.

Film boiling and condensation processes in porous media are relatively new research areas. Contrary to two-phase flows of pure fluids, the study of boiling mixtures in porous media has received little attention. Parmenntier [1] studied film boiling of water about a heated vertical plate in a permeable matrix. Postulating that when boiling occurs adjacent to a vertical surface in a porous medium, a thin vapor film will form, he argued that the vapor film and the liquid were separated by a distinct interface with no mixed region in between. Assuming also that: the density of the subcooled water is constant; the density of the vapor is small compared with that of the saturated water; and that heat conduction in the longitudinal direction is small compared with the transverse direction, Parmenntier obtained an approximate solution for the Nusselt number. An extension of his work was carried out by Cheng [2], who studied film condensation about inclined plates and cones. Cheng, assuming a saturated vapor and using the standard approximations of classical condensation problems, obtained a similarity solution and showed that the Nusselt number decreases with the wall subcooling. Arguing that the governing equations and the boundary conditions were identical to those of film boiling in a porous medium filled with a saturated liquid, Cheng concluded that his solution was applicable to the similar film boiling problem. Later on, Cheng and Verma [3] investigated the effect of subcooling and superheat on the heat transfer rate in film boiling flow about a vertical plate in a permeable matrix. Their analysis showed that the Nusselt number decreased with an increasing superheat but increased with subcooling. Film boiling has also been investigated for other geometrical shapes in porous media. Film boiling processes about spheres and cylinders were analyzed in ref. [4]. This study showed that the heat transfer from the body is enhanced with increasing subcooling, and also by the presence of the porous medium. The predictions for this theoretical work were confirmed later experimentally [5].

Although, two phase flows in porous media are starting to attract more attention, still, the analysis of two phase flow of mixtures has been very much neglected. The research on film boiling or film condensation of mixtures of fluids is rather scarce. Sparrow and Marshall [6] investigated film condensation of a saturated vapor mixture. The results showed that temperature effects in the vapor were negligible. Yue and Weber [7] analyzed film boiling of saturated binary mixtures. Their analysis showed that mixtures with high relative volatilities yield heat fluxes higher than those predicted by Bromley [8] for single components, thus validating the assumption that the omission of diffusion phenomena could lead to erroneous results. As Marshall and Moresco indicated in their analysis of the same problem [9], the reason for erroneous results occurring when the diffusion phenomena are neglected, lies in this interdependence of the interface temperature and the physical properties of the vapor mixtures. This investigation also showed that similar to pure fluids, the Nusselt number decreases with an increasing superheat and increases with an increasing subcooling.

The literature of film boiling of mixtures in porous matrices is almost nonexistent. The aim of this study is, therefore, to develop a predictive theory of film boiling of a binary mixture about a vertical flat plate embedded in a liquid saturated porous medium, and to verify the validity of the theory by comparing the

NOMENCLATURE

a	diffusion coefficient for property G
C	mass fraction of methanol
ΔC	concentration difference
C_p	specific heat
D_c	non-dimensional number defined by equation (43c)
f	dimensionless stream function
G	property to be transported by fluid
g	gravitational acceleration
ΔG	difference of G across the boundary layer
h	heat transfer coefficient
H	length scale
h_{fg}	latent heat
i	current flow
K	permeability of the porous medium
k	thermal conductivity
k_m	thermal conductivity of the fluid and the porous medium
Le	Lewis number
$\dot{m}$	mass flux rate across the interface
N	parameter defined by equation (23)
Nu_x	local Nusselt number, equation (18b)
p	total pressure
q'	heat flux
R	property ratio of the vapor and the liquid phase, $(\rho_\infty/\rho_v)[\mu_v\alpha_L\rho_\infty\beta_T h_{fg}/\mu_L\alpha_v(\rho_\infty-\rho_v)C_{pL}]^{1/2}$
Ra_x	local Rayleigh number
Sc	dimensionless degree of subcooling of liquid, $C_{pL}(T_i-T_\infty)/h_{fg}$
Sh	dimensionless degree of wall superheating, $C_{pv}(T_w-T_i)h_{fg}$
Sw	Sherwood number, equation (27d)
T	temperature
ΔT	temperature difference
U	Darcy's velocity in the x-direction
V	Darcy's velocity in the y-direction
v	voltage
x	coordinate along the surface
y	coordinate perpendicular to the surface.

Greek symbols

α	equivalent thermal diffusivity
β_T, β_c	thermal and concentration coefficient of expansion
Γ	dimensionless G
δ	thickness of boundary layer
ε	infinitesimal temperature variation
η	similarity variable
$\eta_{v\delta}$	dimensionless thickness of the vapor boundary layer
θ	dimensionless temperature
μ	kinematic viscosity
ρ	density
σ	mass diffusion coefficient
ϕ	dimensionless concentration
Ψ	stream function.

Subscripts

0	used to characterize scales
1	methanol
2	water
c	concentration layer
i	interface
j	summation index
L	liquid
p	plate
v	vapor
∞	liquid bulk
*	average.

theoretical results with our own experimental data. The comparison shows good agreement between theory and experiments.

EXPERIMENTAL APPARATUS AND PROCEDURE

The heat transfer measurements made in the present study were performed using the electrically-heated plate and the test section shown in Figs. 1(a) and (b). More specifically, Fig. 1(a) shows the test section. It consists of a rectangular stainless steel box with glass windows on three sides so that visual observations of the phenomenon of interest could be made. The test section was equipped with a vapor condensing section. Care was exercised in maintaining the required volumetric composition of the mixture by not returning the condensate collected in the condensing section back to the test section. The liquid level in the test section was kept constant by means of a large liquid reservoir attached to the test section. Figure 1(b) shows a cross section of the heated plate used for the experiment.

The heated surface consisted of a 38.1 mm by 76.2 mm copper plate and four electric resistance heaters. The heaters were attached to the backside of the copper plate, which was later thermally insulated. Each heater was made out of two 2.54 mm thick high thermal conductivity boron nitride plates and a 0.35 mm diameter nickel–chromium wire. The nickel–chromium resistance wire was connected to a HP Model 3280 data acquisition system. The data acquisition system was used to control the electric power input into the heaters to achieve a uniform temperature at the heated surface. The temperature of each heater was measured with type-k thermocouples (T1–T6) placed at a distance of 0.02 mm below the copper surface that is adjacent to the saturated porous

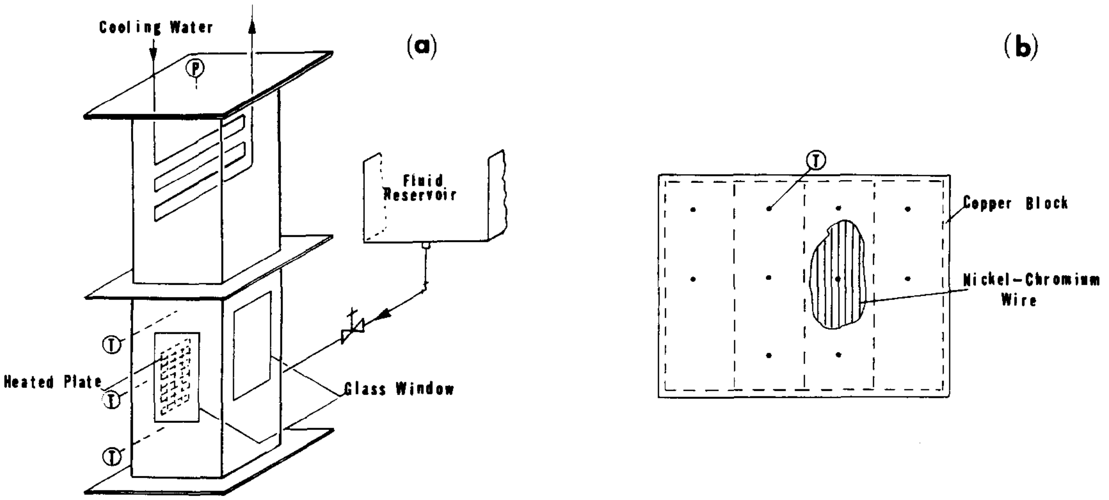

FIG. 1. Test section and boiling surface plate.

medium. A uniform surface temperature was accomplished by reading the thermocouples and comparing their output with the desired temperature. Based on this comparison, the relay of each heater was either opened or closed. The steady-state bulk temperature of the porous medium, too, was measured with three type-k thermocouples, T7, T8, and T9, located directly above, to the side, and directly below the test surface. Steady-state temperature measurements, were made on the heated surface at the rate of one measurement per second for 2 min. The values of voltage and current were time average values since the voltage was switched on and off. The porous medium consisted of 2.8 mm glass beads and the working fluid was a mixture of water and methanol.

Data reduction

For a given power input per heater, the heat flux from the copper surface to the fluid saturated porous bed was calculated from

$$q = \frac{1}{A_{\mathrm{p}}}\sum_{1}^{4}(vi)_j \qquad (1)$$

where v is the voltage across each heater, i the corresponding current flow, and A_{p} the surface area of the copper plate.

Reproducibility between experiments performed at the same condition was quite good, with deviation of only a few percent between comparable data.

The steady-state uncertainty in the heat transfer results was due to observed resolution in electric current readings and in temperature recorded errors. A detailed uncertainty analysis showed that a maximum expected error of ±8% would occur.

MATHEMATICAL FORMULATION

Let us consider the schematic diagram presented in Fig. 2. The physical situation chosen for the study of this problem envisions an isothermal, vertical flat plate embedded in a liquid saturated porous medium. The plate is maintained at a temperature T_{w} which is higher than the saturation temperature of the fluid. The surrounding fluid consists of a mixture of two incompressible and miscible fluids with components which can be evaporated in the temperature range of interest. The vapor layer formed adjacent to the plate surface is a binary mixture of the vapor phases of the evaporating liquids. In the liquid, convective and diffusive motions are induced by two phenomena; first, the interface between the vapor film and the liquid, due to the boiling process, acts as a source of vapor, therefore inducing suction of liquid. On the other hand, density gradients are responsible for natural convection and mass diffusion. The bulk of the liquid is assumed to be at a given temperature T_{∞} corresponding to ambient conditions within a given composition C_{∞}, at atmospheric pressure. The inter-

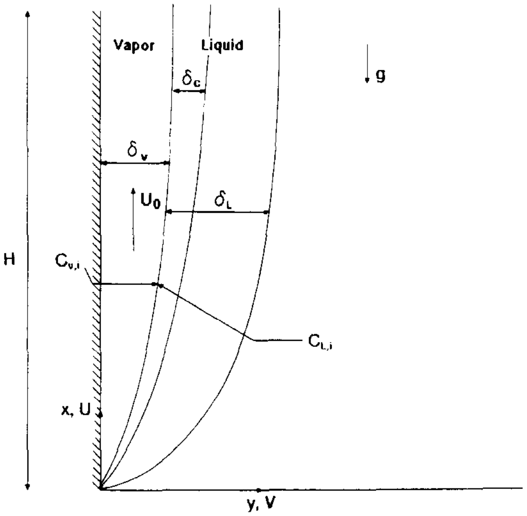

FIG. 2. Physical model.

face between the two phases is at saturation with a constant temperature T_i, the value of which is unknown a priori. Corresponding to T_i and the total pressure p, there are definite values of the interfacial concentrations on both sides of the interface. The knowledge of the temperature T_i is the key to the determination of the heat transfer rate at the interface. In developing a mathematical model for the physical situation described above, the following simplifying assumptions are considered:

(1) A distinct and smooth interface with no regions of mixed phase exists between the vapor and the liquid phase.

(2) Laminar boundary layer approximations are sufficient to describe these flows.

(3) Darcy's flow is applicable in both phases.

(4) The vapor phase and the liquid phase are considered both incompressible.

(5) Boussinesq's approximation is invoked in the liquid phase, so that the density is assumed to be proportional to temperature and concentration.

(6) All other properties of the liquids, the vapors and the medium are constant, and are evaluated at their corresponding average temperatures.

GOVERNING EQUATIONS

With the aforementioned assumptions, the equations governing the steady-state conservation of mass, momentum, energy and species for Darcy flow through a porous medium are given as follows [11].

Vapor phase ($y \leqslant \delta_v$)

$$\frac{\partial U_v}{\partial x}+\frac{\partial V_v}{\partial y}=0 \tag{2}$$

$$U_v=\frac{K}{\mu_v}(\rho_\infty-\rho_v)g \tag{3}$$

$$U_v\frac{\partial T_v}{\partial x}+V_v\frac{\partial T_v}{\partial y}=\alpha_v\frac{\partial^2 T_v}{\partial y^2} \tag{4}$$

$$U_v\frac{\partial C_v}{\partial x}+V_v\frac{\partial C_v}{\partial y}=\sigma_v\frac{\partial^2 C_v}{\partial y^2} \tag{5}$$

where C is the mass fraction of the constituent of interest. In a binary mixture, only one equation for the conservation of species is required since $C_1+C_2=1$. The vapor liquid interface is assumed to be at a uniform temperature T_i, therefore the vapor concentration at the liquid–vapor interface is uniform as well. On the other hand, the impermeability of the wall implies that at that location, the mass flux is zero $\partial C_v/\partial y|_{y=0}=0$. Consequently, in the vapor layer, the concentration is uniform and equation (5) reduces to

$$C_v=C_{v,i}=\text{constant}. \tag{6}$$

The temperature at the interface determines the composition of the fluids in the vapor film.

Liquid phase ($y \geqslant \delta_v$)

$$\frac{\partial U_L}{\partial x}+\frac{\partial V_L}{\partial y}=0 \tag{7}$$

$$U_L=\frac{K\rho_\infty g}{\mu_L}[\beta_T(T_L-T_\infty)+\beta_c(C_L-C_\infty)] \tag{8}$$

$$U_L\frac{\partial T_L}{\partial x}+V_L\frac{\partial T_L}{\partial y}=\alpha_L\frac{\partial^2 T_L}{\partial y^2} \tag{9}$$

$$U_L\frac{\partial C_L}{\partial x}+V_L\frac{\partial C_L}{\partial y}=\sigma_L\frac{\partial^2 C_L}{\partial y^2}. \tag{10}$$

Similarity transformation

A transport equation of the type

$$U\frac{\partial G}{\partial x}+V\frac{\partial G}{\partial y}=a\frac{\partial^2 G}{\partial y^2} \tag{11}$$

may be transformed into the following equation:

$$\Gamma''+\frac{(U_0\delta_0)'\delta_0}{a}\Gamma'=0 \tag{12}$$

by means of the following change of variables:

$$\eta=\frac{y}{\delta_0},\quad f'(\eta)=\frac{u}{U_0},\quad \Gamma=\frac{G}{\Delta G} \tag{13a–c}$$

provided equation (11) describes the transport of a fluid property G in a boundary layer. Here δ_0, U_0 and ΔG represent respectively a reasonable scale for the thickness of the layer, a reasonable scale for the velocity in the layer, and a range of magnitude of property G across that boundary layer. Figure 2 illustrates the two boundary layer problem of the present discussion. From equation (13b) it is possible then to define a stream function Ψ such that

$$\Psi=(U_0\delta_0)f \tag{14}$$

and as a result the x and y components of the velocity are given by

$$U=\frac{\partial\Psi}{\partial y}=U_0f' \tag{15a}$$

$$V=-\frac{\partial\Psi}{\partial x}=-(U_0\delta_0)'f+\eta f'U_0\delta_0'. \tag{15b}$$

As previously indicated U_0 and δ_0 are reasonable scales that can actually be determined by scale analysis.

Bejan [10, 11] has recently shown that for a double diffusion problem the corresponding scales for the flow, temperature, and concentration fields can be determined based on an order of magnitude analysis. For the vapor phase, in a space region defined by δ_v and the plate height H, equation (4) expresses a balance between diffusion of heat across the vapor thickness δ_v and convection along the vapor film. In an order of magnitude sense, the balance convection–conduction yields the following scales for the vapor layer thickness, the vapor velocity, and the Nusselt number, respectively:

$$\delta_{\mathrm{v}} \sim H\, Ra_{\mathrm{v}-H}^{-1/2} \quad (16)$$

$$U_{\mathrm{v}} \sim \frac{\alpha_{\mathrm{v}}}{H} Ra_{\mathrm{v}-H} \quad (17)$$

and

$$Nu_H \sim Ra_{\mathrm{v}-H}^{1/2} \quad (18a)$$

where the local Nusselt number has been defined as

$$Nu_x = \frac{g'}{k\Delta T} \quad (18b)$$

and $Ra_{\mathrm{v}-H}$ is the classical Rayleigh number in the vapor phase based on H and defined by

$$Ra_{\mathrm{v}-H} = \frac{KgH(\rho_\infty - \rho_{\mathrm{v}})}{\mu_{\mathrm{v}}\alpha_{\mathrm{v}}}. \quad (19)$$

Two regions can be identified in the liquid phase as shown in Fig. 2.

(a) The temperature boundary layer ($\delta_{\mathrm{L}} \times H$).
(b) The concentration boundary layer ($\delta_c \times H$).

From equation (8) the x-component of velocity, U_{L}, scales as

$$U_{\mathrm{L}} \sim \frac{\alpha_{\mathrm{L}}}{H} Ra_{\mathrm{L}-H} + N\frac{\alpha_{\mathrm{L}}}{H} Ra_{\mathrm{L}-H} \quad (20)$$

where $Ra_{\mathrm{L}-H}$ is a Rayleigh number based on the height H and defined by

$$Ra_{\mathrm{L}-H} = \frac{K\rho_\infty g\beta_T(T_{\mathrm{i}} - T_\infty)H}{\mu_{\mathrm{L}}\alpha_{\mathrm{L}}} \quad (21)$$

and where N is a non-dimensional number defined by

$$N = \frac{\beta_c(C_{\mathrm{L,i}} - C_\infty)}{\beta_T(T_{\mathrm{i}} - T_\infty)}. \quad (22)$$

An estimate for N can be readily found, that is

$$N \sim \frac{\delta_c}{\delta_{\mathrm{L}}}. \quad (23)$$

From equations (9) and (10) using the same procedure in the derivation of equation (16), we are then able to write

$$\delta_{\mathrm{L}} \sim \left(\frac{\alpha_{\mathrm{L}} H}{U_{\mathrm{L}}}\right)^{1/2} \quad (24a)$$

$$\delta_c \sim \left(\frac{\sigma_{\mathrm{L}} H}{U_{\mathrm{L}}}\right)^{1/2} \quad (24b)$$

which in return implies that the ratio $\delta_{\mathrm{L}}/\delta_c$ depends on the Lewis number defined as $Le = \alpha_{\mathrm{L}}/\sigma_{\mathrm{L}}$. Making use of equations (24a) and (24b) we obtain

$$Le^{1/2} = \frac{\delta_{\mathrm{L}}}{\delta_c}. \quad (25)$$

In our analysis, two cases ought to be considered alternatively in the evaluation of the liquid phase:

(1) The thermal boundary layer is thicker than the concentration layer, that is the Lewis number is greater than one. In this situation, N is smaller than unity which indicates that the flow in the liquid phase is heat transfer driven, and as a result from equations (20), (24a) and (24b) we have

$$U_{\mathrm{L}} \sim \frac{\alpha_{\mathrm{L}}}{H} Ra_{\mathrm{L}-H} \quad (26a)$$

$$\delta_{\mathrm{L}} \sim H\, Ra_{\mathrm{L}-H}^{-1/2} \quad (26b)$$

$$\delta_c \sim H\, Le^{-1/2} Ra_{\mathrm{L}-H}^{-1/2}. \quad (26c)$$

(2) If the thermal boundary layer is much thinner than the concentration layer ($Le \ll 1$), the flow is mass driven, and as a result, equations (20), (24a) and (24b) yield

$$U_{\mathrm{L}} \sim \frac{\alpha_{\mathrm{L}}}{H} Ra_{\mathrm{L}-H} Le^{-1/2} \quad (27a)$$

$$\delta_c \sim H\, Le^{-1/4} Ra_{\mathrm{L}-H}^{-1/2} \quad (27b)$$

$$\delta_{\mathrm{L}} \sim H\, Ra_{\mathrm{L}-H}^{-1/2} Le^{1/4} \quad (27c)$$

$$Sw \sim Ra_{\mathrm{L}-H}^{1/2} Le^{1/4} \quad (27d)$$

where Sw is the Sherwood number. Thus the ability to estimate the mass transfer scale reduces to being able to estimate the concentration layer thickness δ_c.

SIMILARITY SOLUTION

Vapor phase

Equations (16) and (17) may now be chosen such that

$$U_0 = \frac{\alpha_{\mathrm{v}}}{x} Ra_{\mathrm{v}-x} \quad (28a)$$

and

$$\delta_0 = x\, Ra_{\mathrm{v}-x}^{-1/2} \quad (28b)$$

then equations (13a)–(13c) result in

$$\eta_{\mathrm{v}} = \frac{y}{x} Ra_{\mathrm{v}-x}^{1/2}, \quad f'_{\mathrm{v}}(\eta_{\mathrm{v}}) = \frac{U_{\mathrm{v}}}{U_0} = 1, \quad \theta_{\mathrm{v}} = \frac{T - T_{\mathrm{w}}}{T_{\mathrm{i}} - T_{\mathrm{w}}} \quad (29a\text{–}c)$$

whereas equations (14) and (15b) become

$$\Psi_{\mathrm{v}} = \alpha_{\mathrm{v}} Ra_{\mathrm{v}-x}^{1/2} \quad (30)$$

and

$$V_{\mathrm{v}} = \frac{\alpha_{\mathrm{v}}}{2x} Ra_{\mathrm{v}-x}^{1/2}[\eta_{\mathrm{v}} f'_{\mathrm{v}} - f_{\mathrm{v}}] \quad (31)$$

and finally equation (12) yields

$$\theta''_{\mathrm{v}} + \tfrac{1}{2} f_{\mathrm{v}} \theta'_{\mathrm{v}} = 0. \quad (32)$$

Liquid phase

Following the same procedure, U_0 and δ_0 may be chosen from either equations (26a) and (26b) or (27a) and (27c). It is interesting to note that the scale given by both sets of equations will lead to a similarity solution. However, disregarding the order of magnitude of the Lewis number may lead to 'non-reasonable' scales, thus misleading one in the appropriate interpretation of the results. In our experiments it was observed that for a mixture of water and methanol, the Lewis number was always much greater than unity, therefore the scales given by equations (26a) and (26b) were chosen. We then define

$$\eta_L = \frac{y-\delta_v}{\delta_0}, \quad f'_L = \frac{U_L}{U_0} \tag{33a,b}$$

$$\theta_L = \frac{T_L - T_\infty}{T_i - T_\infty}, \quad \phi_L = \frac{C_L - C_\infty}{C_{L,i} - C_\infty}. \tag{33c,d}$$

In terms of equation (12), equations (9) and (10) now become

$$\theta''_L + \tfrac{1}{2} f_L \theta'_L = 0 \tag{34}$$

$$\phi''_L + \frac{Le}{2} f_L \phi'_L = 0. \tag{35}$$

Next, the parameter N can be expressed by

$$N = \frac{\Delta C}{\Delta T} \frac{\partial T/\partial y}{\partial C/\partial y} \tag{36}$$

and equation (8) becomes

$$f'_L = (\phi'_L \theta_L + \phi_L \theta'_L)/\phi'_L. \tag{37}$$

Boundary conditions

At the wall

$$\begin{cases} T_v(y=0) = T_w \\ V_v(y=0) = 0 \end{cases} \text{or} \quad \begin{cases} \theta_v(\eta_v = 0) = 0 \\ f_v(\eta_v = 0) = 0. \end{cases} \tag{38a,b}$$

In the bulk of the liquid

$$\begin{cases} T_L(y \to \infty) = T_\infty \\ C_L(y \to \infty) = C_\infty \end{cases} \text{or} \quad \begin{cases} \theta_L(\eta_L \to \infty) = 0 \\ \phi_L(\eta_L \to \infty) = 0. \end{cases} \tag{39a,b}$$

At the liquid–vapor interface

$$\begin{cases} T_v(y = \delta_v) = T_i \\ T_L(y = \delta_v) = T_i \\ C_L(y = \delta_v) = C_{L,i} \end{cases} \text{or} \quad \begin{cases} \theta_v(\eta_v = \eta_{v\delta}) = 1 \\ \theta_L(\eta_L = 0) = 1 \\ \phi_L(\eta_L = 0) = 1. \end{cases} \tag{40a–c}$$

At the liquid–vapor interface the following three conditions must also be satisfied:

(a) Conservation of mass

$$\rho_v(U_v\delta'_v - V_v) = \rho_L(U_L\delta'_v - V_L) = \dot{m} \tag{41a}$$

or in its non-dimensional form

$$\eta_{v\delta} = R(Sc)^{1/2} f_L(0) \tag{41b}$$

where the following quantities have been introduced:

$Sc = C_{pL}(T_i - T_\infty)h_{fg}$ which is a measure of the degree of subcooling as defined by Cheng and Verma [3] and

$$R = \frac{\rho_\infty}{\rho_v}\left[\frac{\rho_v \alpha_L \rho_\infty \beta_T h_{fg}}{\mu_L \alpha_v (\rho_\infty - \rho_v) C_{pL}}\right]^{1/2} \tag{42}$$

a non-dimensional number containing all the physical properties of the problem.

(b) Conservation of species

$$\rho_{1,v}(U_v\delta'_v - V_v) + \rho_v\sigma_v\frac{\partial C_v}{\partial y} = \rho_{1,L}(U_L\delta'_v - V_L) + \rho_L\sigma_L\frac{\partial C_L}{\partial y} \tag{43a}$$

which in terms of the similarity variable reduces to

$$D_c\phi'_L(0) = -\frac{Le}{2} f_L(0) \tag{43b}$$

where D_c is defined by

$$D_c = \frac{C_\infty - C_{L,i}}{C_{v,i} - C_{L,i}}. \tag{43c}$$

(c) Conservation of energy

$$-k_{m,v}\left.\frac{\partial T_v}{\partial y}\right|_{y=\delta_v} = mh_{fg} - k_{m,L}\left.\frac{\partial T_L}{\partial y}\right|_{y=\delta_v} \tag{44a}$$

which reduces to

$$Sh\,\theta'_v(\eta_{v\delta}) + R\theta'_L(0)(Sc)^{3/2} = \frac{\eta_{v\delta}}{2} \tag{44b}$$

where $Sh = C_{pv}(T_w - T_i)/h_{fg}$ is a measure of the degree of superheat present in the vapor film [3].

In the vapor layer, equation (32) with boundary conditions (38a), (38b) and (40a) yield an exact solution

$$\theta'_v = \frac{e^{-(\eta_v/2)^2}}{\sqrt{\pi}\,\mathrm{erf}\,(\eta_{v\delta}/2)} \tag{45}$$

$$\theta_v = \frac{\mathrm{erf}\,(\eta_v/2)}{\mathrm{erf}\,(\eta_{v\delta}/2)}. \tag{46}$$

In terms of the similarity variables, the heat transfer coefficient h and the Nusselt number are given by

$$h = [k_{m,L}/x\, Ra_{v-x}^{-1/2}]\theta'_v(0) \tag{47}$$

and

$$Nu_x\, Ra_{v-x}^{-1/2} = \theta'_v(0) = \frac{1}{\sqrt{\pi}\,\mathrm{erf}\,(\eta_{v\delta}/2)} \tag{48}$$

respectively.

Similarly, a measure of the mass diffusion rate across the interface is given by

$$Sw\, Ra_{L-x}^{-1/2} = -\phi'_L(0). \tag{49}$$

NUMERICAL SOLUTION

The analytical formulation is now to be solved numerically for a mixture of water and methanol at 24°C at a system pressure of 1 atm. The major unknown of this problem is the interface temperature. As soon as it is determined, a predictive theory for the effect of concentration, subcooling, and superheat on the heat transfer rate can be derived following the procedure used by Cheng and Verma [3]. The value of the interface temperature T_i does not depend on the wall temperature T_w or on the reservoir temperature T_∞. It solely depends on the composition of the liquid in the reservoir. Therefore, our major concern in this analysis is to describe the relation between C_∞ and T_i. Five unknowns can be identified in this problem, namely T_i, $\eta_{v\delta}$, $f_L(0)$, $\theta'_L(0)$ and $\phi'_L(0)$. The concentration in the vapor is constant and equal to the concentrations of the vapor at the interface, which is determined by the interface temperature T_i.

For each phase and each compound, the physical properties were evaluated first at an average temperature and concentration. The empirical correlations used for the computations of the physical properties are given by Reid and Sherwood [13]. The density of the mixtures and the latent heat were computed as suggested by Sparrow and Marshall [6]. The coefficient of thermal expansion and the specific heat of the mixtures were calculated based on a mass average. Finally, once the thermal conductivities of the liquid and the vapor were obtained, the analysis of Orozco *et al.* [5] was used to determine the thermal conductivity of the porous medium. The saturation state diagram of a boiling mixture of water and methanol is given in a discretized form by Perry [14]. Four polynomials of order five were used to describe the concentration/temperature relationship for the mixture of water and methanol.

The numerical solution of this problem is found in two steps. First, equations (34), (35) and (37), are integrated using boundary conditions (40b) and (40c). The unknown boundary conditions $\theta'_L(0)$, $\phi'_L(0)$ and $f_L(0)$ are determined by a shooting technique to match the conditions set by equations (38a), (38b) and (43b). Note that in this process, the Lewis number Le, and D_c are the parameters characterizing the fluid and the conditions of the boiling process. Thus, the first step consists in integrating the governing equations in the liquid phase as described above for different pairs (Le, D_c). It is worth mentioning that for any given Lewis number, solutions are available only in the range $0 \leqslant D_c < 1$. Let us recall from equation (43c) that

$$D_c = \frac{C_\infty - C_{L,i}}{C_{v,i} - C_{L,i}}. \tag{43c}$$

From the diagram of the saturation state, Fig. 3, $C_{v,i} - C_{L,i}$ is a positive quantity. On the other hand, due to the boiling process, the concentration of methanol at the interface on the liquid side is smaller than the concentration of methanol in the bulk of the liquid, therefore $C_\infty - C_{l,i}$ is also a positive quantity, and $D_c > 0$. The value $D_c = 0$ refers to a situation where $C_{v,i} \neq C_{l,i}$ and $C_\infty = C_{L,i}$, that is, a situation where the bulk of the liquid is at saturation state and the temperature of the liquid is the same as the interface temperature. In this case there is neither temperature nor concentration gradients in the liquid region. In order to understand the second limit ($D_c < 1$) let us consider the diagram in Fig. 3 once more.

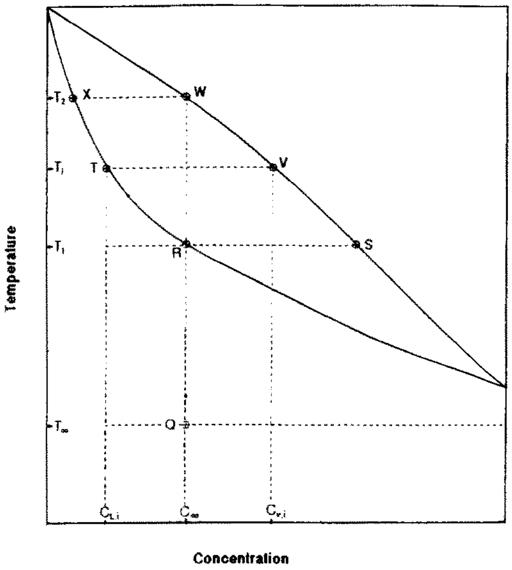

FIG. 3. Schematic development of interface conditions.

Prior to the immersion of the plate, the bulk of the fluid is at temperature T_∞ with a composition C_∞ denoted by point Q. The immersion of the plate causes a rise of temperature from T_∞ to where boiling occurs. Provided that the fluid is not at saturation state, the temperature in the control volume rises until it reaches the final temperature T_i. In this process the concentration of the liquid in the neighborhood of the liquid vapor interface decreases along the line R–T, while the concentration of the vapor decreases along the line S–V. If further heating was to be experienced the interface temperature would rise to T_2 where the liquid is completely vaporized to give a vapor of the exact same composition as the original liquid (point W). In the analysis we assume an infinite supply of liquid in the reservoir. Therefore, the interface temperature has to be between T_1 and T_2. Note that at point W, $C_\infty = C_{v,i}$, in other words $D_c = 1$ and at point R, $C_\infty = C_{L,i}$, in other words $D_c = 0$. Therefore, the limits of D_c appear to be an indication of the limits of the interface temperature for a given mixture. This observation leads to the second step of our numerical analysis. The energy equation (44b) is rewritten as

$$F(T_i) = Sh\,\theta'_v(\eta_{v\delta}) + R\,\theta'_L(0)(Sc)^{3/2} - \frac{\eta_{v\delta}}{2} = 0. \tag{50}$$

For a given concentration C_∞, D_c is evaluated for the temperature $T_1 + \varepsilon$ and $T_2 - \varepsilon$, where ε is a small temperature differential. The physical properties are then computed at $T_i = (T_1 + T_2)/2$. The Lewis number obtained is used to integrate equations (34), (35) and (37) with the boundary conditions given by equations (40b), (40c), (39a), (39b) and (43). The major results of this integration are $\theta'_L(0)$, $\phi'_L(0)$ and $f_L(0)$ which are determined by the shooting technique. Equation (41b) gives $\eta_{v\delta}$ and the functions $F(T_i + \varepsilon)$ and $F(T_2 - \varepsilon)$ are evaluated. When it is found that the product $F(T_i+\varepsilon)\times F(T_2-\varepsilon)$ is negative, a dichotomic process is used and the same procedure is repeated until $F(T_i)=0$.

RESULTS AND DISCUSSION

Theoretical results

The first observation to be made is that the interface temperature does not depend on the ambient temperature, nor does it depend on the wall temperature. The liquid–vapor interface temperature depends exclusively on the mixture composition in the reservoir. Therefore, a mixture with a given composition shows the exact same film boiling behavior as any other fluid with a fixed boiling temperature. The analysis of the effect of subcoold liquid on film boiling can then be performed for a particular mixture following the procedure used by Cheng and Verma [3].

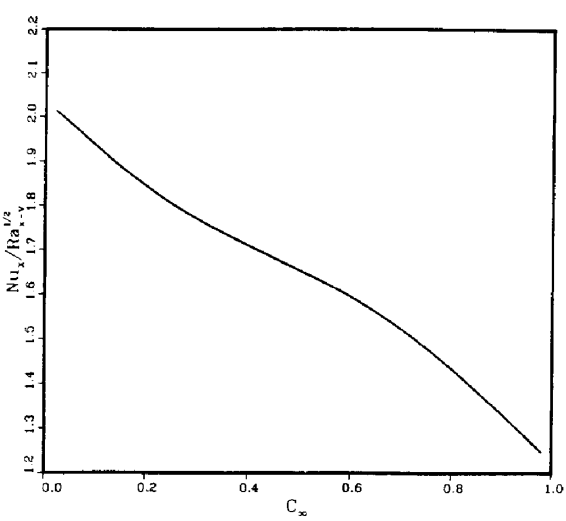

FIG. 5. Quality of heat transfer rate with mixture composition.

The first result to be discussed is the relationship between the interface temperature T_i and the reservoir mass fraction (see Fig. 4). As expected, the interface temperature of the mixture lies somewhere between the boiling temperature of each compound. For a given concentration of methanol in the liquid, there corresponds a given interface temperature and mass fractions on both sides of the interface. In the present analysis, Fig. 5 shows that the Nusselt number decreases with an increase of methanol concentration. In other words, we reach the same conclusion that Marshall and Moresco [9] reached for pure fluids, that is, the Nusselt number increases with increasing water concentration in a water–methanol mixture. The reason for that as we can see from Fig. 4, is that an increase in methanol concentration results in a decrease of the interface temperature. Therefore, the superheat increases and the Nusselt number decreases, this behavior, familiar in classical film boiling problems, strengthens the idea that film boiling of pure fluids or liquid mixtures is very similar. It can also be noted from Fig. 6 that the thickness of the vapor layer increases with the concentration of methanol. As stated before, the increase of concentration results in an increase of superheat. Therefore, one concludes that the thickness of the vapor layer increases with the superheat. This result is in agreement with the conclusions of Cheng and Verma's analysis [3]. Figure 6 also indicates that the diffusion phenomena cannot be neglected in the evaluation of the heat transfer rate, the reason is that the diffusion phenomenon imposes the interface temperature. Figure 7 indicates that the Sherwood number, a measure of mass diffusion, increases with the concentration of methanol. For an increase in the proportion of methanol in the liquid, the proportion of methanol diffusing through the interface increases. Figures 8 and 9 show the dimensionless temperature and concentration profile in the liquid for various bulk mass fractions. They indicate

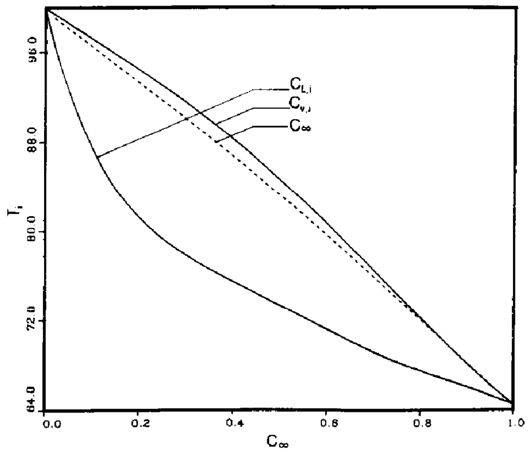

FIG. 4. Effect of bulk concentration on the interface temperature.

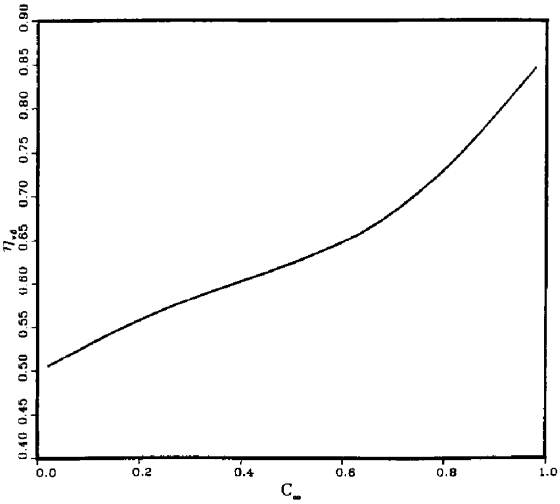

FIG. 6. Influence of reservoir composition on vapor thickness.

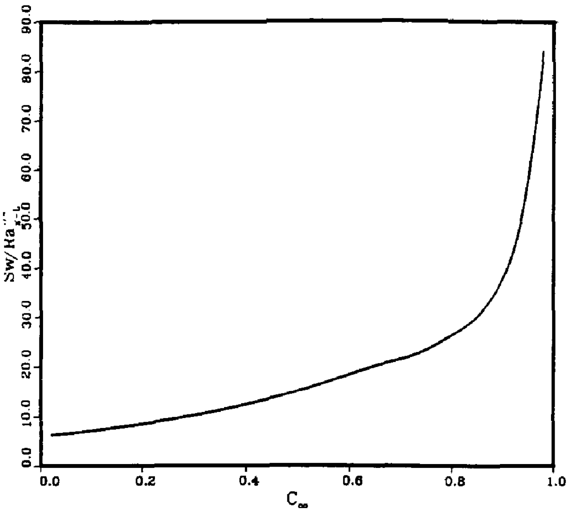

FIG. 7. Quality of mass transfer rate with mixture composition.

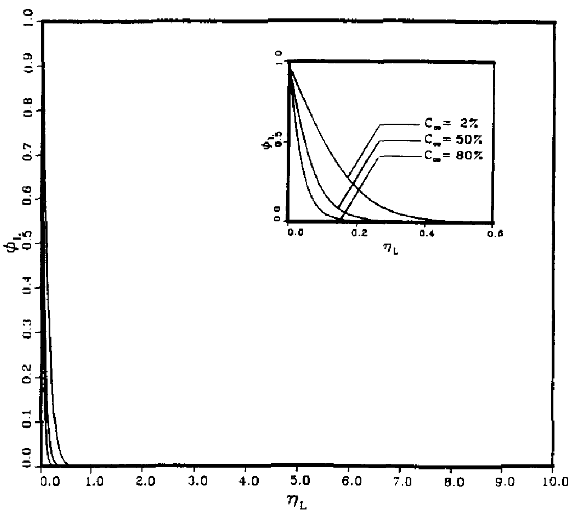

FIG. 9. Dimensionless mixture concentration profiles.

that the corresponding boundary layers become narrower as the concentration in the bulk increases. Note also how stiff the concentration profile is. This is due to the large Lewis number (*Le* = 40). Figure 10 presents the dimensionless temperature in the vapor for various C_∞. The profile is quasi linear and does not vary with the reservoir concentration.

Experimental results and comparison to theory

Figure 10 is a comparison of the theory developed in this investigation to our own measurements for film boiling from a vertical plate embedded in a liquid saturated porous medium. The comparison of the theory to the experimental data shows a good agreement between theory and experiments. The theory underpredicts the experimental data by 20%. This discrepancy may be attributed to the wavy nature of the vapor film as well as to the evaluation of physical properties, especially the thermal conductivity of the liquid saturated porous medium.

CONCLUSION

The theoretical analysis has shown that film boiling of binary mixtures is very similar to film boiling of pure fluids. As a matter of fact, a mixture with a composition C_∞ can be viewed as a particular pure fluid with a fixed liquid–vapor interface temperature. In that respect, the analysis of the effect of subcooling and superheat on the heat transfer rate can be predicted theoretically based on the procedure developed in ref. [3]. Diffusion in the liquid cannot be neglected,

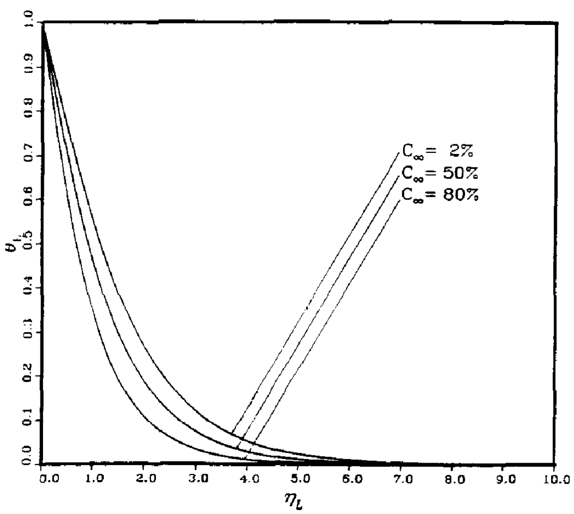

FIG. 8. Dimensionless liquid temperature distribution as function of mixture bulk composition.

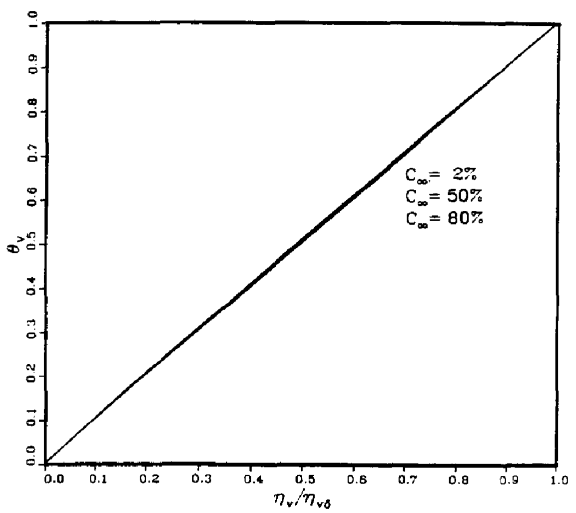

FIG. 10. Dimensionless vapor film temperature profiles as a function of mixture bulk temperature.

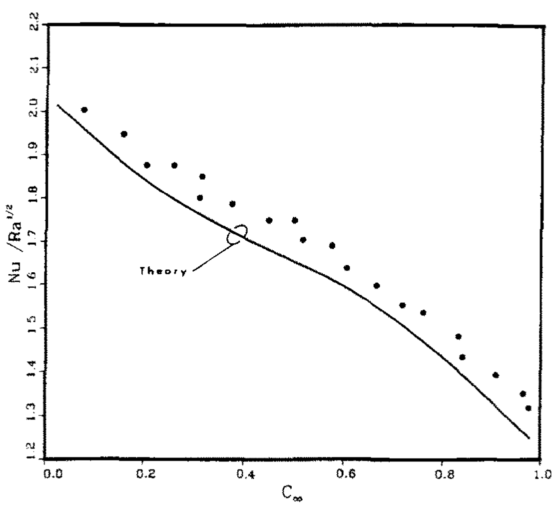

FIG. 11. Comparison between theory and experimental for a methanol/water mixture at 24°C.

since it controls the interface temperature. The theory developed in this investigation predicts the heat flux very well for the heat transfer regime when stable film boiling exists over the entire area of a vertical plate embedded in a liquid-saturated porous medium. Good agreement exists between the theory and our own experimental data.

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UNE ANALYSE DE L'EBULLITION EN FILM D'UN MELANGE BINAIRE DANS UN MILIEU POREUX

Résumé—Une analyse théorique concerne l'effet des phénomènes de diffusion sur le mécanisme de l'ébullition en film pour une plaque verticale et un mélange liquide dans un milieu poreux. Avec les approximations de couche limite, les lois de conservation sont résolues à l'aide de transformations de similitude. Le système obtenu d'équations différentielles est résolu en utilisant une technique de tir. Des résultats de transfert thermique sont obtenus pour un domaine de conditions opératoires correspondant à un mélange d'eau et de méthanol. Des expériences sont conduites pour vérifier les résultats théoriques et un excellent accord est trouvé entre la théorie et l'expérience.

UNTERSUCHUNG DES FILMSIEDENS EINES BINÄREN GEMISCHES IN EINEM PORÖSEN MEDIUM

Zusammenfassung—Der Einfluß von Diffusionsvorgängen auf den Wärmeübergang bei Filmsieden an einer senkrechten Platte, die an ein mit einem Flüssigkeitsgemisch gesättigtes poröses Medium grenzt, wird theoretisch untersucht. Unter Verwendung der Grenzschichtvereinfachungen und mit Hilfe von Ähnlichkeitstransformationen werden die Erhaltungsgleichungen gelöst. Dabei ergibt sich ein Satz von Differentialgleichungen, der schließlich mit einem Zufallsverfahren ausgewertet wird. Für ein Wasser/Methanol-Gemisch wird in einem Bereich von Betriebsbedingungen der Wärmeübergang ermittelt. Dabei zeigt sich, daß die Wärmeübergangskoeffizienten mit wachsender Methanolkonzentration abnehmen. Zur Bestätigung der theoretischen Ergebnisse werden ergänzend Versuche durchgeführt—die Übereinstimmung ist hervorragend.

АНАЛИЗ ПЛЕНОЧНОГО КИПЕНИЯ БИНАРНОЙ СМЕСИ В ПОРИСТОЙ СРЕДЕ

Аннотация—Теоретически анализируется влияние явлений диффузии на процесс теплопереноса от вертикальной пластины к жидкой смеси в пористой среде в условиях пленочного кипения. С использованием приближения пограничного слоя уравнения сохранения решаются методом преобразований подобия. Полученная система дифференциальных уравнений решается методом пристрелки. Найдены результаты, характеризующие теплоперенос в широком диапазоне изменения режимных параметров применительно к смеси воды и метанола. Проведены эксперименты, результаты которых очень хорошо согласуются с теоретическими выводами.